

## The Origin of Risk

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What drives individual risk-taking decisions and how do they affect aggregate risk?

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- Because of endogenous risk, **distortions** can make **GDP more volatile**

We **calibrate** the model to the Spanish economy

- Removing distortions lead to a large decline in aggregate volatility

Most of macroeconomics takes risk as **exogenous** (at the micro and/or macro level)

- In **models with individual firms**, firm-level risk is generally exogenous but macro risk can be endogenous
  - Khan and Thomas (2008), Clementi and Palazzo (2016), Bloom et al. (2018), and many others
- In **endogenous growth models**, firms influence the growth rate of TFP but not its variance
  - Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Jones (1995)
- **Growth models** in which more complete markets push firms to adopt high-risk, high-reward projects,
  - Greenwood and Jovanovic (1990), Acemoglu and Zilibotti (1997), Cole et al. (2016)
- **Corporate finance** literature where managers influence how risky a project is
  - Jensen and Meckling (1976), Ross (1977)
- **Wedges in production network economies**
  - Jones (2011), Baqaee and Farhi (2019), Liu (2019) and Bigio and La'O (2020)
- **Technique choice in production networks**
  - Oberfield (2018), Acemoglu and Azar (2020), Kopytov et al. (2024)



## A model of endogenous risk

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Static model with two types of agents

1. A **representative household** owns the firms, supplies labor and risk management resources
2.  $N$  **firms** produce differentiated goods using labor and intermediate inputs
  - Firm  $i$  has constant returns to scale **Cobb-Douglas production function**

$$F(\delta_i, L_i, X_i) = e^{a_i(\varepsilon, \delta_i)} \zeta_i L_i^{1 - \sum_{j=1}^N \alpha_{ij}} \prod_{j=1}^N X_{ij}^{\alpha_{ij}}$$

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There are **underlying sources of risk**  $\boldsymbol{\varepsilon} = (\varepsilon_1, \dots, \varepsilon_M)$  with  $\boldsymbol{\varepsilon} \sim \mathcal{N}(\boldsymbol{\mu}, \boldsymbol{\Sigma})$

- Examples: a river floods, discovery of a new drug, war between two countries, epidemic, etc.
- We don't take a stance on what  $\boldsymbol{\varepsilon}$  is. Focus on **quantity of risk** and **correlation** structure.

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Managing risk (picking  $\boldsymbol{\delta}_i$ ) requires **risk management resources**  $R_i$  supplied by the household

$$R_i = \kappa_i(\boldsymbol{\delta}_i) = \frac{1}{2} (\boldsymbol{\delta}_i - \boldsymbol{\delta}_i^\circ)^\top H_i (\boldsymbol{\delta}_i - \boldsymbol{\delta}_i^\circ)$$

where  $\boldsymbol{\delta}_i^\circ$  is the *natural* risk exposure ( $R_i = 0$ ), and  $H_i$  is a positive definite matrix

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Maximizes King, Plosser, Rebelo (1988) preferences

$$\mathcal{U}(Y) \mathcal{V}(R)$$

where  $\mathcal{U}$  is CRRA with risk aversion  $\rho \geq 1$ , and disutility of risk management  $\mathcal{V}(R)$  is

► Details

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Budget constraint in each state of the world (set  $W_L = 1$  from now on)

$$\sum_{i=1}^N P_i C_i \leq W_L + W_R R + \Pi$$

## Timing

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Cobb-Douglas **unit cost** is

$$K_i(\delta_i, P) = \frac{1}{e^{a_i(\varepsilon, \delta_i)}} \prod_{j=1}^N P_j^{\alpha_{ij}}$$

## Risk-taking decision

Firm choose their risk exposure to maximize **expected discounted profits**

$$\delta_i^* \in \arg \max_{\delta_i} E [\Lambda [P_i Q_i - K_i (\delta_i, P) Q_i - \kappa_i (\delta_i) W_R]]$$

where  $Q_i$  is *equilibrium* demand and  $\Lambda$  is the **stochastic discount factor** of the household.

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$$\delta_i^* \in \arg \min_{\delta_i} \underbrace{\mathbb{E} [K_i (\delta_i, P) Q_i]}_{(1)} + \underbrace{\text{Cov} \left( K_i (\delta_i, P) Q_i, \frac{\Lambda}{\mathbb{E}[\Lambda]} \right)}_{(2)} + \underbrace{\kappa_i (\delta_i) W_R}_{(3)}$$

Firms prefer risk exposures  $\delta_i$  with

1. high expected TFP (low expected unit costs  $K_i$ )
2. low covariance with GDP = low correlation with GDP + low variance of unit cost  $K_i$ 
  - Rely on less volatile risk factors, or **diversify** by using offsetting risk factors
3. low risk management expenses  $\kappa_i (\delta)$

## Equilibrium definition

An *equilibrium* is a risk choice for every firm  $\delta^*$  and a stochastic tuple  $(P^*, W_R^*, C^*, L^*, R^*, X^*, Q^*)$  such that

1. (Optimal technique choice) For each  $i$ , factor demand  $L_i^*$ ,  $X_i^*$  and  $R_i^*$ , and the risk exposure decision  $\delta_i^*$  solves the firm's problem.
2. (Consumer maximization) The consumption vector  $C^*$  and the supply of risk managers  $R^*$  solve the household problem.
3. (Unit cost pricing) For each  $i$ ,  $P_i = (1 + \tau_i) K_i(\delta_i, P)$ .
4. (Market clearing) For each  $i$ ,

$$C_i^* + \sum_{j=1}^N X_{ji}^* = Q_i^* = F_i(\alpha_i^*, L_i^*, X_i^*), \quad \sum_{i=1}^N L_i^* = 1, \quad \text{and} \quad \sum_{i=1}^N \kappa_i(\delta_i^*) = R^*.$$



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Revenue-based Domar weight:

$$\omega^T = \beta^T \mathcal{L} = \beta^T (I - [\text{diag}(1 + \tau)]^{-1} \alpha)^{-1}$$

- Also captures importance as a supplier (share of revenues)
- Declines with **wedges**  $\tau$

Define **aggregate risk exposure**  $\Delta$  as

$$\Delta := \delta^\top \tilde{\omega}$$

- Firms with high cost-based Domar weights contribute more to aggregate risk exposure

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### Lemma

$$\log Y = y = \Delta^\top \varepsilon - \tilde{\omega}^\top \log(1 + \tau) - \log(\text{Labor share}(\omega, \tau))$$

- Without distortions ( $\tau = 0$ ) we have Hulten's theorem:  $y = \Delta^\top \varepsilon = \omega^\top a(\varepsilon, \delta)$

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### Impact of $\Sigma$

- A marginal increase in  $\Sigma_{mm}$  raises  $V[y]$  by  $\Delta_m^2$ 
  - Both  $\Delta_m \gg 0$  and  $\Delta_m \ll 0$  are bad for  $V[y]$
- If the economy is positively exposed to  $m$  and  $n$ , increasing  $\Sigma_{mn}$  raises  $V[y]$ .
- If  $\Delta_m > 0$  and  $\Delta_n < 0$ , the shocks offset each other. Higher  $\Sigma_{mn}$  reduces  $V[y]$ .

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## Impact of $\Delta$

$$\frac{dV[y]}{d\Delta_m} = 2 \text{Cov}[y, \varepsilon_m] = 2 \sum_n \Delta_n \text{Cov}[\varepsilon_n, \varepsilon_m]$$

- Extra exposure to  $\varepsilon_m$  increases volatility if  $\varepsilon_m$  is positively correlated with GDP



## Existence, uniqueness and efficiency

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## Planner's problem

Define  $\bar{\kappa}_{SP}(\Delta)$  as the **smallest risk management utility cost** needed to achieve  $\Delta$ .

$$\bar{\kappa}_{SP}(\Delta) := \min_{\delta} -\log V\left(\sum_{i=1}^N \kappa_i(\delta_i)\right), \quad \text{subject to } \Delta = \delta^\top \tilde{\omega}$$

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### Planner's problem

$$\mathcal{W}_{SP} := \max_{\Delta} \underbrace{\Delta^\top \mu}_{\mathbb{E}[y_{SP}]} - \frac{1}{2}(\rho - 1) \underbrace{\Delta^\top \Sigma \Delta}_{\mathbb{V}[y_{SP}]} - \bar{\kappa}_{SP}(\Delta)$$

The planner prefers aggregate risk exposure vectors  $\Delta$  with

- high expected GDP  $\mathbb{E}[y_{SP}]$
- low GDP volatility  $\mathbb{V}[y_{SP}]$
- low risk management cost  $\bar{\kappa}_{SP}$

## Equilibrium characterization through fictitious planner

Define  $\bar{\kappa}(\Delta)$  as the **perceived** smallest risk management utility cost needed to achieve  $\Delta$ .

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Proposition (fictitious planner's problem)

There exists a **unique equilibrium**, and it solves

$$\mathcal{W}_{dist} := \max_{\Delta} \underbrace{\Delta^\top \mu - \tilde{\omega}^\top \log(1+\tau) - \log \Gamma_L}_{\mathbb{E}[y]} - \frac{1}{2} (\rho - 1) \underbrace{\Delta^\top \Sigma \Delta}_{\mathbb{V}[y]} - \bar{\kappa}(\Delta).$$

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The equilibrium solves a **distorted planning problem**

- Still seeks to maximize  $\mathbb{E}[y]$  and minimize  $\mathbb{V}[y]$
- But **distorted perception** of the cost of managing risk ( $\bar{\kappa}$  instead of  $\bar{\kappa}_{SP}$ )

## Determinants of equilibrium risk

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## Lemma

The equilibrium aggregate risk exposure  $\Delta$  solves

$$\underbrace{\mathcal{E}(\Delta)}_{\text{marginal benefit of } \Delta} = \underbrace{\nabla \bar{\kappa}(\Delta)}_{\text{marginal cost of } \Delta}$$

where the **marginal value of aggregate risk exposure**  $\mathcal{E}$  is given by

$$\mathcal{E} = \mathbb{E}[\varepsilon] + \text{Cov}[\lambda, \varepsilon],$$

and where  $\lambda = \log \Lambda$  is the log of the stochastic discount factor.



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Equation for  $\mathcal{E}$  implies that firms **prefer risk factors** with

- high expected value  $\mu = \mathbb{E}[\varepsilon]$  and negative covariance with GDP ( $\text{Cov}[\lambda, \varepsilon] > 0$ )
- Risk factor is “good” if  $\mathcal{E} > 0$  and “bad” if  $\mathcal{E} < 0$

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Role of **risk attractiveness**  $\mathcal{E}$  and **risk management technology**  $H_i$

- Higher  $\mathcal{E}_m$  always leads to higher  $\delta_{im}$
- Higher  $\mathcal{E}_m$  can increase or decrease  $\delta_{in}$ 
  - **Local complements** if  $H_{i,mn}^{-1} > 0$ :  $\delta_{im}$  and  $\delta_{in}$  tend to move together
  - **Local substitutes** if  $H_{i,mn}^{-1} < 0$ :  $\delta_{im}$  and  $\delta_{in}$  tend to move in opposite directions

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Role of **firm size** as measured by cost of goods sold  $K_i Q_i \propto \omega_i / (1 + \tau_i)$

- TFP multiplies the input bundle
- Risk management benefit grows with  $K_i Q_i$  while its cost  $\kappa_i W_R$  does not
- **High Domar weight**  $\omega_i$  and **low wedge**  $\tau_i$  firms **manage risk more aggressively**

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The equilibrium aggregate risk exposure decisions are

$$\Delta = \Delta^\circ + \mathcal{H}^{-1} \mathcal{E}^\circ,$$

where  $\mathcal{E}^\circ = \mathcal{E}(\Delta^\circ)$  and where the  $M \times M$  positive definite matrix  $\mathcal{H}^{-1}$  is

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Role of **natural risk attractiveness**  $\mathcal{E}^\circ = \mathcal{E}(\Delta^\circ) = \mu - (\rho - 1) \Sigma \Delta^\circ$

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Global substitution patterns depend on

- $\nabla^2 \bar{\kappa}$ : global impact of the **local substitution patterns** embedded in  $(\kappa_1, \dots, \kappa_N)$
- $\Sigma$ : if  $\Sigma_{mn} > 0$  an increase in  $\Delta_m$  makes the planner reduce  $\Delta_n$  to avoid agg. risk

### Proposition

Let  $\gamma$  be either  $\mu_m$  or  $\Sigma_{mn}$ . Then

$$\frac{d\Delta}{d\gamma} = \mathcal{H}^{-1} \frac{\partial \mathcal{E}}{\partial \gamma}$$

- The vector  $\partial \mathcal{E} / \partial \gamma$  captures the *direct impact* of  $\gamma$  on the attractiveness of risk factors ▶  $\partial \mathcal{E} / \partial \gamma$
- The matrix  $\mathcal{H}^{-1}$  *propagates* that impact to exposure vector  $\Delta$



### Proposition

Let  $\gamma$  be either  $\mu_m$  or  $\Sigma_{mm}$ . Then

$$\frac{d\Delta}{d\gamma} = \mathcal{H}^{-1} \frac{\partial \mathcal{E}}{\partial \gamma}$$

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- The matrix  $\mathcal{H}^{-1}$  *propagates* that impact to exposure vector  $\Delta$

### Corollary

1. An increase in  $\mu_m$  raises  $\Delta_m$
  2. An increase in  $\Sigma_{mm}$  reduces  $\Delta_m$  if  $\Delta_m > 0$  and increases  $\Delta_m$  if  $\Delta_m < 0$
- A marginal increase in  $\Sigma_{mm}$  raises  $V[y]$  by  $\Delta_m^2 \rightarrow$  When  $\Sigma_{mm}$  increases we want to *reduce*  $\Delta_m^2$

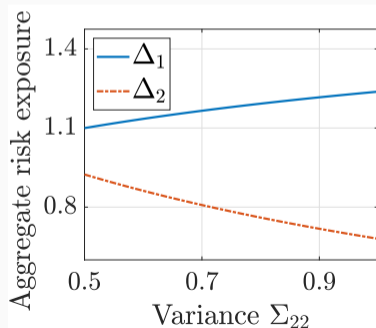
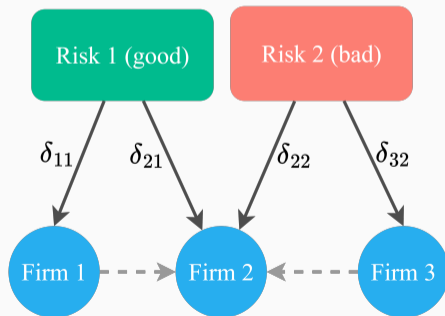
## Example of substitution patterns

There are **two regions** both with their specific shocks

- Region 1: more productive in expectation (Risk 1 – good risk)
- Region 2: bigger shocks (Risk 2 – bad risk)

Firm 2 must decide **where to locate plants**

- Challenging to manage plants in different locations → risks are substitutes



**Definition.** An economy is diagonal if  $\Sigma$  and  $H_i$  are diagonal for every  $i$

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## Corollary

In a diagonal economy, a higher wedge  $\tau_i$

1. increases  $\Delta_m$  for all  $m$  such that  $\mathcal{E}_m < 0$  (bad risks)
2. reduces  $\Delta_m$  for all  $m$  such that  $\mathcal{E}_m > 0$  (good risks)

- Higher wedges make firms shrink  $\rightarrow$  manage risk less aggressively

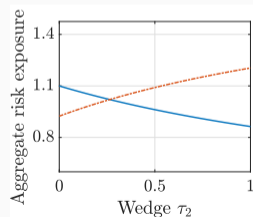
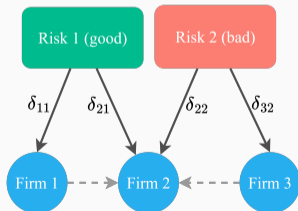
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(Blue: good risk; Red: bad risk)

When all firms are at their **natural exposure**  $\delta^\circ$  we have  $\mathcal{E}^\circ = \mu - (\rho - 1) \Sigma \Delta^\circ$

Lemma

Equilibrium risk exposure is distorted such that  $(\Delta - \Delta_{SP})^\top \mathcal{E}^\circ < 0$ .

When all firms are at their **natural exposure**  $\delta^\circ$  we have  $\mathcal{E}^\circ = \mu - (\rho - 1) \Sigma \Delta^\circ$

### Lemma

Equilibrium risk exposure is distorted such that  $(\Delta - \Delta_{SP})^\top \mathcal{E}^\circ < 0$ .

- Wedges make firms **inefficiently small**  $\rightarrow$  less risk management
- Eqm. is on average **overexposed to bad risks** ( $\mathcal{E}^\circ < 0$ ) and **underexposed to good risks** ( $\mathcal{E}^\circ > 0$ )

## Implications for GDP and Welfare

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Use  $\partial$  to denote changes in the economy with **exogenous risk**

### Proposition

In a diagonal economy:

$$\text{sign} \left( \frac{d \mathbf{E}[y]}{d \mu_m} - \frac{\partial \mathbf{E}[y]}{\partial \mu_m} \right) = \text{sign}(\mu_m) \quad \text{and} \quad \frac{d \mathbf{V}[y]}{d \Sigma_{mm}} - \frac{\partial \mathbf{V}[y]}{\partial \Sigma_{mm}} < 0.$$

- Increasing  $\mu_m$  raises  $\Delta_m \rightarrow$  additional increase in  $\mathbf{E}[y]$  if  $\mu_m > 0$  compared to fixed risk
- Increasing  $\Sigma_{mm}$  decreases  $|\Delta| \rightarrow$  smaller increase in  $\mathbf{V}[y]$  than with fixed risk

### Proposition (single risk factor)

$$\text{sign} \left( \frac{d\mathbf{E}[y]}{d\tau_i} - \frac{\partial \mathbf{E}[y]}{\partial \tau_i} \right) = -\text{sign}(\mu\mathcal{E}) \quad \text{and} \quad \text{sign} \left( \frac{d\mathbf{V}[y]}{d\tau_i} - \frac{\partial \mathbf{V}[y]}{\partial \tau_i} \right) = -\text{sign}(\Delta\mathcal{E}).$$

Suppose  $\mathcal{E} < 0$  (bad risk, e.g. business cycle): increasing  $\tau_i$  makes firms more exposed to risk factor

- if  $\mu < 0$  this leads to a decline in  $\mathbf{E}[y]$
- if  $\Delta > 0$  the economy becomes even more exposed and  $\mathbf{V}[y]$  increases

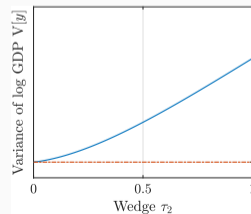
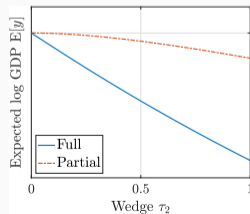
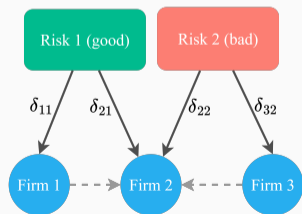
# Distortions can increase aggregate volatility

## Proposition (single risk factor)

$$\text{sign} \left( \frac{dE[y]}{d\tau_i} - \frac{\partial E[y]}{\partial \tau_i} \right) = -\text{sign}(\mu\mathcal{E}) \quad \text{and} \quad \text{sign} \left( \frac{dV[y]}{d\tau_i} - \frac{\partial V[y]}{\partial \tau_i} \right) = -\text{sign}(\Delta\mathcal{E}).$$

Suppose  $\mathcal{E} < 0$  (bad risk, e.g. business cycle): increasing  $\tau_i$  makes firms more exposed to risk factor

- if  $\mu < 0$  this leads to a decline in  $E[y]$
- if  $\Delta > 0$  the economy becomes even more exposed and  $V[y]$  increases



### Proposition

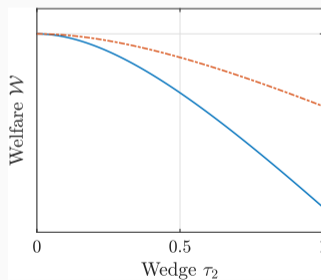
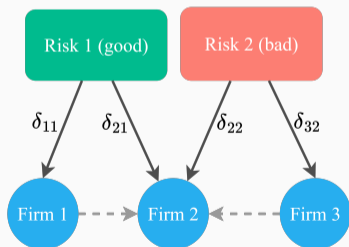
In a diagonal economy, raising  $\tau_i$  hurts welfare more than under exogenous risk.

- A higher  $\tau_i$  increases exposure to bad risks and lower exposure to good risks
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(Blue: flexible risk; Red: fixed risk)

## Reduced-form evidence

---

Model: firms with large Domar weights and small markups are less volatile and less corr. with GDP

▶ Details

Model: firms with **large Domar weights** and **small markups** are **less volatile** and **less corr. with GDP**

► Details

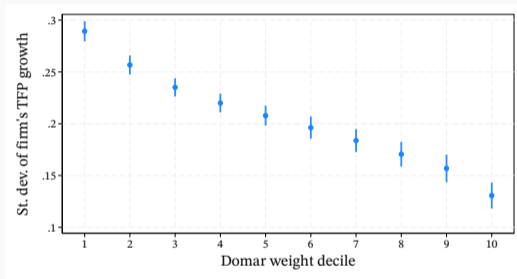
We test these predictions in the data

- Use detailed **micro data** from the near-universe of firms in **Spain** between 1995 and 2018 (Orbis) (7,513,081 firm-year observations)
- Compute **markups** using control function approach (De Loecker and Warzynski, 2012)
- Back out TFP growth as a residual

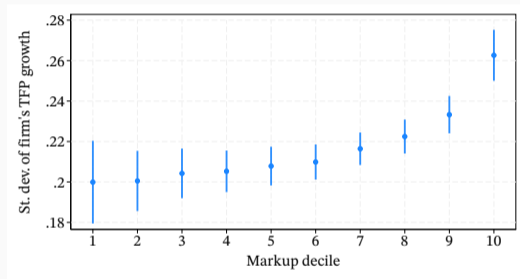
► Details



# TFP growth volatility



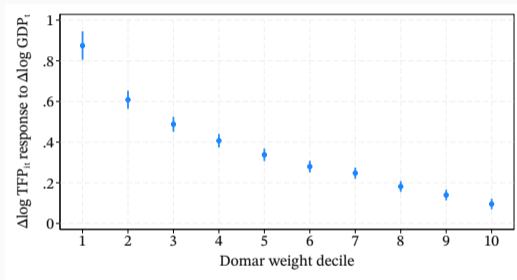
(a) TFP volatility by Domar weight decile



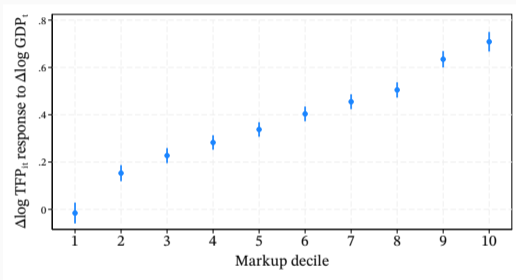
(b) TFP volatility by markup decile

► Details

# Covariance of TFP growth with GDP growth



(c) Sensitivity of firm TFP to GDP by Domar weight decile



(d) Sensitivity of firm TFP to GDP by markup decile

► Details

## Calibration

---

## A specialized model to map to the data

- **S sectors** with aggregator  $Q_s = \prod_{i=1}^{N_s} e^{z_s} (\theta_{si}^{-1} Q_{si})^{\theta_{si}}$  and sectoral shocks  $z_s \sim \text{iid } \mathcal{N}(\mu_s^z, \Sigma_s^z)$
- Firms have **production function**

$$Q_{si} = e^{\delta_{sit}\varepsilon_t + \gamma_{sit}t + v_{sit}} \zeta_{si} L_{si}^{1 - \sum_{s'} \hat{\alpha}_{ss'}} \prod_{s'=1}^S X_{si,s'}^{\hat{\alpha}_{ss'}}$$

where  $\hat{\alpha}_{ss'}$  are sectoral shares,  $v_{sit} \sim \text{iid } \mathcal{N}(\mu_{si}^v, \Sigma_{si}^v)$  and  $\varepsilon_t \sim \text{iid } \mathcal{N}(0, \Sigma)$

- Risk management **cost function** is parametrized as

$$\frac{1}{\eta} H_{si}^{-1} = a_s \tilde{\omega}_{si}^{b_s} + c_s$$

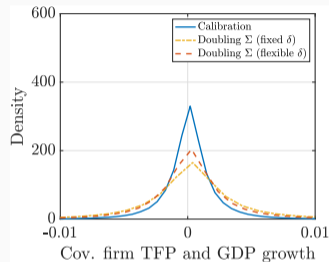
Allows for a **size effect** on risk management costs

► Details

- We aim at **replicating** as much of the firm-level Spanish data as possible
- Our calibrated model has 62 sectors and 492,917 individual firms
- We invert parts of the model to **exactly match some moments**
  1. Sectoral consumption shares and input/output cost shares
  2. Firm shares in sectoral sales
  3. Variance of firm TFP growth
  4. Covariance of firm TFP growth and GDP growth
  5. Variance of GDP growth

What if we double the volatility  $\Sigma$  of the risk factor?

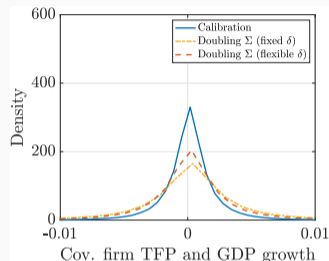
	Calibration	Doubling $\Sigma$	
		Fixed $\delta$	Flexible $\delta$
Agg. risk exposure $\Delta$	0.014	0.014	0.011
Exposure value $\mathcal{E}$	-0.06	-0.11	-0.09
Std. Dev. of GDP growth	2.4%	3.1%	2.6%



- **Fixed  $\delta$** : Large increase in **GDP variance**; exposure to  $\varepsilon_t$  becomes more harmful ( $\mathcal{E}$  declines)
- **Flexible  $\delta$** : Firms manage risk more aggressively which **limits increase in  $V[y]$**

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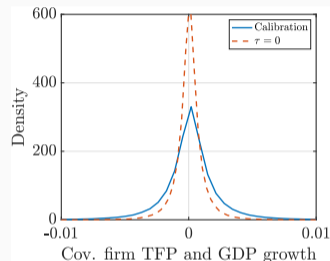
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Impact of risk can be overestimated if reaction of agents is not taken into account

## Removing distortions

What if we set wedges  $\tau$  to zero?

	Calibration	No wedges	
		Fixed $\delta$	Flexible $\delta$
Agg. risk exposure $\Delta$	0.014	0.014	0.007
Exposure value $\mathcal{E}$	-0.06	-0.06	-0.03
Std. Dev. of GDP growth	2.4%	2.4%	1.7%



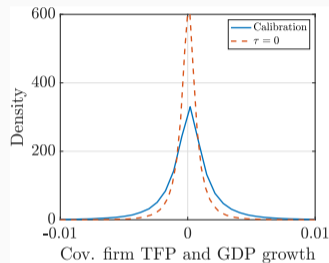
- **Fixed  $\delta$** : Since only impact of  $\tau$  is through  $\delta$ , there is no change.
- **Flexible  $\delta$** : Firms manage risk more aggressively so  $V[y]$  declines



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Distortions can make GDP more volatile

## Conclusion

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## Main contributions

- We construct a model of **endogenous risk**, at both the micro and macro levels.
- Model predicts which firms are more volatile and covary more with business cycle.
- Distortions lead to less aggressive risk management and can increase GDP volatility.

## Future research

- What if there are entrepreneurs who cannot diversify their risk?
- Mechanisms would interact with capital/investment. Fully dynamic business cycle model.

The function  $\zeta(\alpha_i)$  is

$$\zeta(\alpha_i) = \left[ \left( 1 - \sum_{j=1}^n \alpha_{ij} \right)^{1 - \sum_{j=1}^n \alpha_{ij}} \prod_{j=1}^n \alpha_{ij}^{\alpha_{ij}} \right]^{-1}$$

This functional form allows for a simple expression for the unit cost  $K$

Given the log-normal nature of uncertainty  $\rho \leq 1$  determines whether the agent is risk-averse or not. To see this, note that when  $\log C$  normally distributed, maximizing

$$\mathbf{E} [C^{1-\rho}]$$

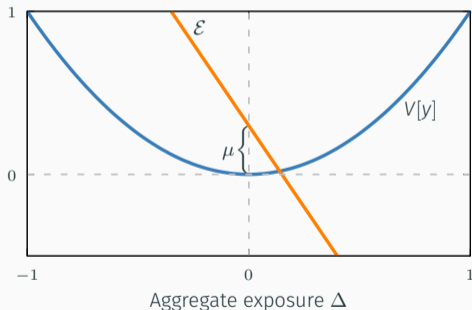
amounts to maximizing

$$\mathbf{E} [\log C] - \frac{1}{2} (\rho - 1) \mathbf{V} [\log C].$$

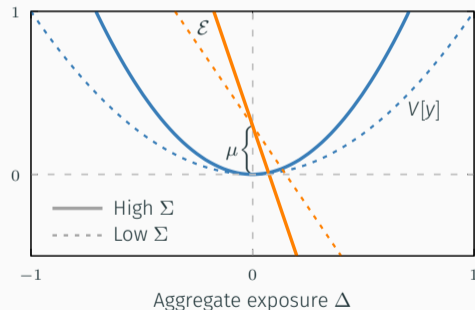
The direct impact of changes in  $(\mu, \Sigma)$  is given by

$$\frac{\partial\mathcal{E}}{\partial\mu_m} = \mathbf{1}_m \quad \text{and} \quad \frac{\partial\mathcal{E}}{\partial\Sigma_{mn}} = -\frac{1}{2}(\rho - 1)(\Delta_m\mathbf{1}_n + \Delta_n\mathbf{1}_m).$$

(a) Low  $\Sigma$



(b) High  $\Sigma$



## Proposition

The response of the equilibrium aggregate risk exposure  $\Delta$  to a change in wedge  $\tau_i$  is given by

$$\frac{d\Delta}{d\tau_i} = \mathcal{T} \left( \sum_{j=1}^N \frac{\partial [\nabla^2 \bar{\kappa}]^{-1}}{\partial g_j} \frac{dg_j}{d\tau_i} \right) \mathcal{E}, \quad (1)$$

where the impact of  $g_j$  on  $[\nabla^2 \bar{\kappa}]^{-1}$  is given by  $\frac{\partial [\nabla^2 \bar{\kappa}]^{-1}}{\partial g_j} = -\frac{1}{\eta} \frac{\tilde{\omega}_j^2}{g_j^2} H_j^{-1}$ , and where

$$\mathcal{T} := \left( I - [\nabla^2 \bar{\kappa}]^{-1} \frac{\partial \mathcal{E}}{\partial \Delta} \right)^{-1}.$$

## Proposition

Let  $\chi$  denote either  $\mu_m$ ,  $\Sigma_{mn}$ , or  $\tau_i$ . Then the impact of a change in  $\chi$  on the moments of log GDP are given by

$$\frac{dE[y]}{d\chi} - \frac{\partial E[y]}{\partial\chi} = \mu^\top \frac{d\Delta}{d\chi} \quad \text{and} \quad \frac{dV[y]}{d\chi} - \frac{\partial V[y]}{\partial\chi} = 2\Delta^\top \Sigma \frac{d\Delta}{d\chi},$$

where the use of a partial derivative indicates that  $\Delta$  is kept fixed.



## Simplified model

◀ Back

- Single risk factor  $\varepsilon_t \sim \text{iid } \mathcal{N}(0, \Sigma)$
- Firm level TFP is  $\log TFP_{it} = \delta_{it}\varepsilon_t + \gamma_i t + v_{it}$  where  $\gamma_i$  is deterministic trend and  $v_{it} \sim \text{iid } \mathcal{N}(\mu_i^v, \Sigma_i^v)$

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[← Back](#)

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## Variance of firm-level TFP growth

$$V[\log TFP_{it} - \log TFP_{it-1}] = 2\delta_i^2 \Sigma + 2\Sigma_i^v$$

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[← Back](#)

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$$V[\log TFP_{it} - \log TFP_{it-1}] = 2\delta_i^2 \Sigma + 2\Sigma_i^v$$

## Covariance of firm-level TFP growth with GDP growth

$$\text{Cov}[\log TFP_{it} - \log TFP_{it-1}, y_t - y_{t-1}] = 2\Delta \Sigma \delta_i + 2\tilde{\omega}_i \Sigma_i^v.$$

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$$\text{Cov}[\log TFP_{it} - \log TFP_{it-1}, y_t - y_{t-1}] = 2\Delta \Sigma \delta_i + 2\tilde{\omega}_i \Sigma_i^v.$$

## Model-implied firm risk exposure ( $\mathcal{E} < 0$ )

$$\delta_i = \delta_i^o + \frac{1}{\eta} \frac{\omega_i}{1 + \tau_i} H_i^{-1} \mathcal{E}$$

⇒ Firms with large Domar weights and small markups are less volatile and less corr. with GDP

- Assume Cobb-Douglas production function

$$\log Q_{it} = \alpha_{L_i} \log L_{it} + \alpha_{M_i} \log M_{it} + \alpha_{K_i} \log K_{it} + \varepsilon_{it},$$

- Elasticities estimated using Levinsohn and Petrin (2003) with the Ackberg et al. (2015) correction.
  - Capital is the “state” variable, labor is the “free” variable and materials is the “proxy” variable.
- Production function estimated at NACE 2-digit sector level. As in De Loecker et al. (2020), we control for markups using firms’ sales shares in the production function estimation.
- Following De Loecker and Warzynski (2012), we compute the markup as  $1 + \tau_{it} = \hat{\alpha}_{L_i} / \left( \frac{\text{Wage Bill}_{it}}{\text{Sales}_{it}} \right)$ .
- We compute TFP growth as

$$\begin{aligned} \Delta \log \text{TFP}_{it} = & \Delta \log Q_{it} - \alpha_{L_i} \Delta \log L_{it} - \alpha_{M_i} \Delta \log M_{it} - \alpha_{K_i} \Delta \log K_{it} \\ & - \left( \Delta \log (1 + \tau_{it}) - \Delta \log (1 + \tau_{s(i)t}) \right). \end{aligned}$$

The term  $\Delta \log (1 + \tau_{it}) - \Delta \log (1 + \tau_{s(i)t})$  accounts for the firm-specific markup growth net of the sectoral markup growth. This adjustment allows us to remove the change in firm-specific nominal price that are not taken into account by the sector-level price deflator.

- We compute the standard deviation of TFP growth for each firm,  $\sigma_i (\Delta \log TFP_{it})$ , and the time-series average of its markup and Domar weight.
- We construct deciles based on average Domar weights and markups, and create dummy variables,  $FE_{ji}^{Domar}$  and  $FE_{ji}^{Markup}$ , such that  $FE_{ji}^{Domar} = 1$  if firm  $i$ 's Domar weight is in decile  $j$ , and analogously for markups.
- We run the cross-sectional regression

$$\sigma_i (\Delta \log TFP_{it}) = \alpha + \sum_{j=1}^{10} \beta_j^{Domar} FE_{ji}^{Domar} + \sum_{j=1}^{10} \beta_j^{Markup} FE_{ji}^{Markup} + \varepsilon_i,$$

and plot  $\beta_j^{Domar}$  in panel (a) and  $\beta_j^{Markup}$  in panel (b).

- We construct deciles based on firms' Domar weights and markups each year.
- We then construct a set of dummy variables,  $FE_{jit}^{Domar}$  and  $FE_{jit}^{Markup}$ , such that  $FE_{jit}^{Domar} = 1$  if firm  $i$ 's Domar weight is in decile  $j$  in year  $t$ , and analogously for markups.
- We then run the following panel regression,

$$\begin{aligned}\Delta \log TFP_{it} = & \sum_{j=1}^{10} \beta_j^{Domar} \left( FE_{jit}^{Domar} \times \Delta \log GDP_t \right) + \sum_{j=1}^{10} \beta_j^{Markup} \left( FE_{jit}^{Markup} \times \Delta \log GDP_t \right) \\ & + \alpha + \beta_0 \Delta \log GDP_t + \sum_{j=1}^{10} FE_{jit}^{Domar} + \sum_{j=1}^{10} FE_{jit}^{Markup} + \varepsilon_{it},\end{aligned}$$

where  $\Delta \log TFP_{it}$  is the annual growth of firm  $i$ 's log TFP and  $\Delta \log GDP_t$  is the annual growth of Spanish log GDP.

- The coefficients of interest,  $\beta_j^{Domar}$  and  $\beta_j^{Markup}$ , are reported in the figure.

- Risk exposure

$$\delta_{si} = \delta_{si}^{\circ} + \frac{\tilde{\omega}_{si}}{g_{si}} \left( \frac{1}{\eta} H_{si}^{-1} \right) \mathcal{E}$$

- The variance of GDP growth is

$$V[y_t - y_{t-1}] = 2\Sigma\Delta^2 + 2\tilde{\omega}_f^{\top}\Sigma^v\tilde{\omega}_f + 2\tilde{\omega}_s^{\top}\Sigma^z\tilde{\omega}_s.$$

- The variance of firm-level TFP growth is

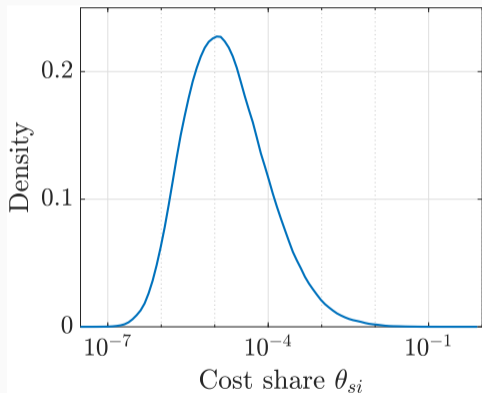
$$V[\log TFP_{si,t} - \log TFP_{si,t-1}] = 2\delta_{si}^2\Sigma + 2\Sigma_{si}^v.$$

- The covariance of firm-level TFP growth with GDP growth is

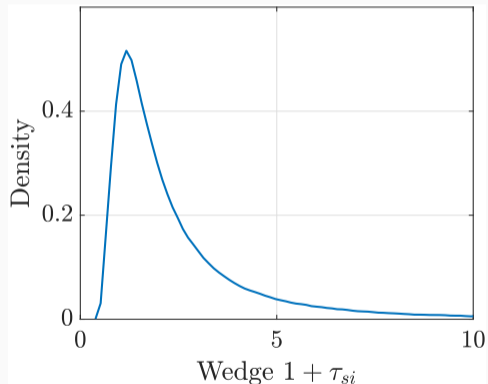
$$\text{Cov}[y_t - y_{t-1}, \log TFP_{si,t} - \log TFP_{si,t-1}] = 2\Delta\Sigma\delta_{si} + 2\tilde{\omega}_{si}\Sigma_{si}^v.$$



Figure 1: Data distributions that the calibration matches exactly

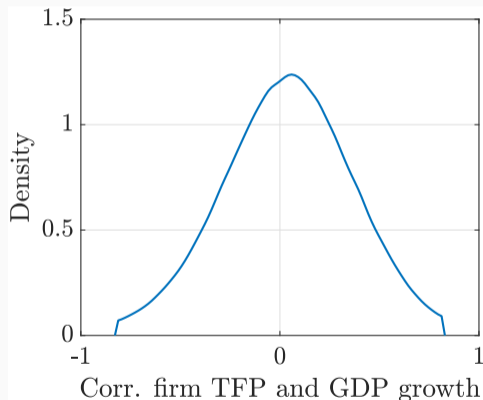


(a) Sales share  $\theta_{si}$

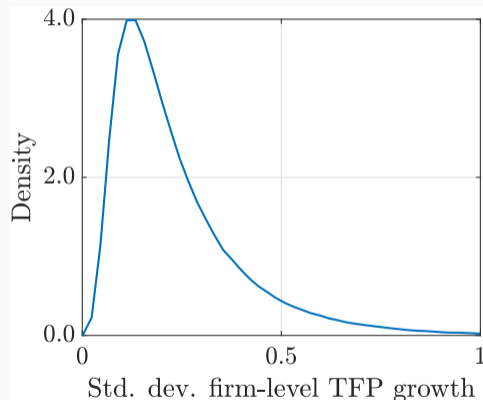


(b) Wedges  $1 + \tau_i$

Figure 2: Data distributions that the calibration matches exactly

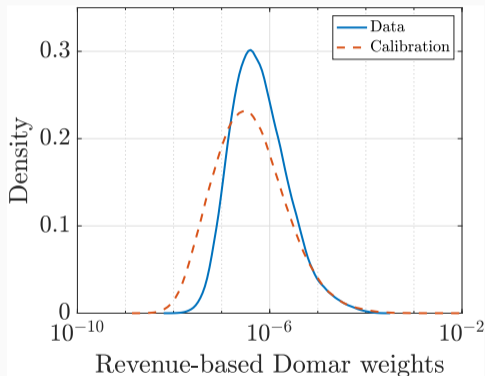


(a) Correlation firm-level TFP and GDP growth

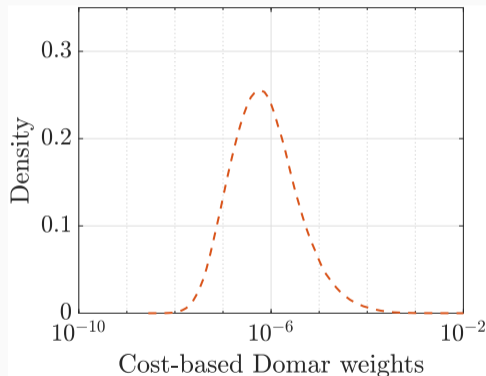


(b) Standard deviation of firm-level TFP growth

Figure 3: Domar weights of the firms in the data and in the model

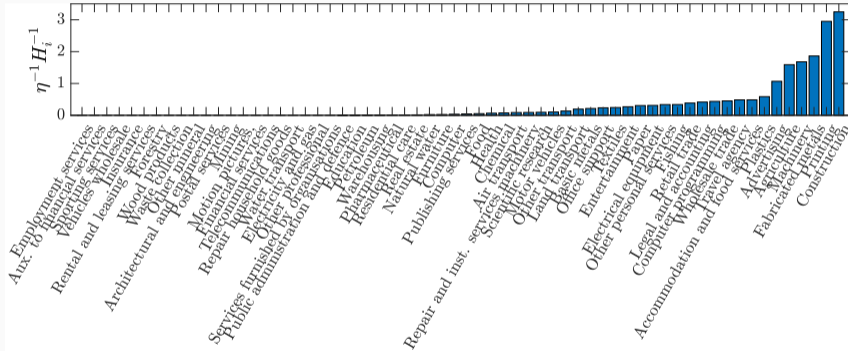


(a) Revenue-based Domar weights



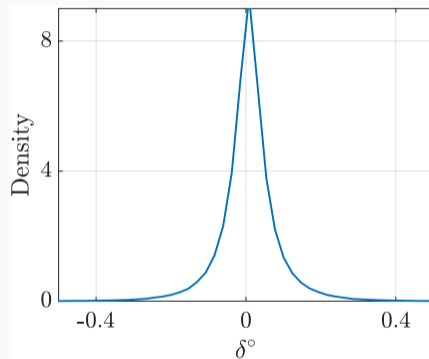
(b) Cost-based Domar weights

Figure 4: Estimated value of  $\frac{1}{\eta} H_i^{-1}$  for each sector.



Notes. The scale of  $\frac{1}{\eta} H_i^{-1}$  depends on our choice of  $\rho$  and  $\Sigma$ . We set  $\rho = 5$  and  $\Sigma = 1$  for this figure.

Figure 5: Distribution of the estimated firm-level natural risk exposure  $\delta_i^\circ$



Notes. The scale of  $\delta_i^\circ$  depends on our choice of  $\rho$  and  $\Sigma$ . We set  $\rho = 5$  and  $\Sigma = 1$  for this figure.